# Wall Layer of Plane Turbulent Wall Jets without Pressure Gradients

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## Theme

THIS paper presents friction coefficients and velocity profiles in the wall layer of constant-pressure turbulent wall jets (two-dimensional jets tangent to the test section floor) for local maximum to mainstream velocity ratios, $U_m/U_e$, from about 1.0 to $\infty$.

## Contents

Experiments with jet to mainstream velocity ratios, $U_j/U_e$ from 1.5 to $\infty$, were conducted in a two-dimensional wall-jet facility with jet slot heights of 0.056 and 0.155 in. over a span of 30 in. For cases with mainstream flow, the mainstream velocity $U_e$ was $\approx 120$ fps. Velocity profiles were computed from measured pressure and temperature profiles. Experimental friction coefficients were determined using Preston tubes.

The experimental velocity profiles on the semilogarithmic plot of $U^+$ vs $\log Y^+$ [where $U^+ = u/u_\tau$, $Y^+ = u_\tau y/v$, $u_\tau = (\tau_w/\rho)^{1/2}$] converge to a common line near the wall. This linear portion has a 15% lower slope than Coles' "law of the wall" for boundary layers.[1] At $\log Y^+ = 2.3$, $U^+$ agree closely with Coles' equation. (In the foregoing definitions, $u$ is local velocity, $y$ is distance from the wall, $v$ is kinematic viscosity, $\tau_w$ is wall shear stress, and $\rho$ is density.)

The data were analyzed assuming that the velocity profiles approached Coles' equation for the law of the wall. The corresponding friction velocities, $u_\tau$, and hence the friction coefficients, were evaluated by demanding a systematic approach to the law of the wall, considering all velocity ratios. For large $U_m/U_e$, the velocities deviated from Coles' equation nearly linearly with $y$. For low $U_m/U_e$ ($U_m/U_e \lesssim 2.5$) the velocity profiles asymptotically approach Coles' equation. For $U_m/U_e$ near 1.0, a boundary-layer-type wake region is formed in the wall layer. It is believed that pressure measurement errors due to turbulence effects and Preston tube errors account for the fact that all the experimental data yielded a common semilogarithmic region of reduced slope (compared to Coles' equation) near the wall.

The nearly linear correlation of the velocity deviations from Coles' law of the wall, as evaluated from

$$U^+ = 5.616 \log Y^+ + 5 + \Delta u/u_\tau \qquad (1)$$

is illustrated in Fig. 1 for $U_j/U_e = \infty$. $y$ is arbitrarily normalized by the thickness $\delta$ for which $\Delta u/u_\tau = -16$ rather than the wall-layer thickness $\delta_m$ (corresponding to the location of the maximum velocity, $U_m$) for better accuracy. Using the empirical correlation equation for $\Delta u/u_\tau$, the velocity profile becomes

$$U^+ = 5.616 \log Y^+ + 5 - 33.3 \operatorname{erf}[0.0652(y/\delta_m)] \qquad (2)$$

Equation (2) evaluated at $y = \delta_m$ gives

$$(2/C_f)^{1/2} = 2.808 \log C_f + 5.616 \log Re_{\delta_m} + 1.709 \qquad (3)$$

where $Re_{\delta_m} = u_m \delta_m / v$ and $C_f = \tau_w / \rho U_m^2$. Equation (3) gives $C_f$ values which are 11% higher than those obtained from the Preston tube data.

Equation (2) yields a velocity defect law ($u - U_m/u_\tau$ vs $y/\delta_m$) which agrees well with the experimental data to $y = \delta_{1/2}$ ($\delta_{1/2}$ is the distance from the wall to the location in the outer layer where $u = U_{m/2}$). Equation (2) may also be rearranged to express $u/U_m$ in terms of $y/\delta_m$ or $y/\delta_{1/2}$. For example, Eq. (2) gives

$$\frac{u}{U_m} = \frac{1}{2} + \left(\frac{C_f}{2}\right)^{\frac{1}{2}} \left\{ 33.3 \left[ \operatorname{erf}\left(0.0652 \frac{\delta_{1/2}}{\delta_m}\right) - \operatorname{erf}\left(0.0652 \frac{\delta_{1/2}}{\delta_m} \frac{y}{\delta_{1/2}}\right)\right] + 5.616 \log \frac{y}{\delta_{1/2}} \right\} \qquad (4)$$

where $\delta_{1/2}/\delta_m$ is evaluated by setting $u/U_m = 1$ at $y = \delta_m$ and $C_f$ is given by Eq. (3). Equation (4) is compared with the experimental data in Fig. 2.

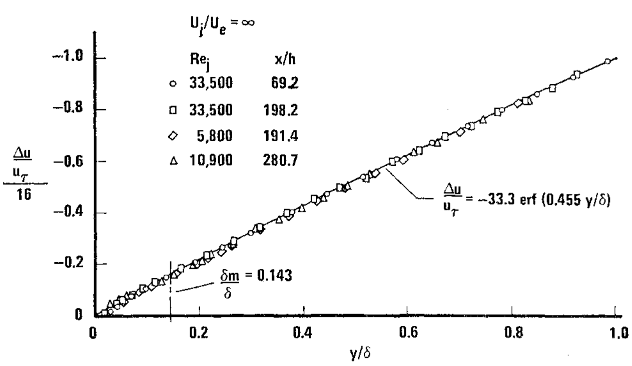


**Fig. 1 Similarity plot of velocity deviation.**

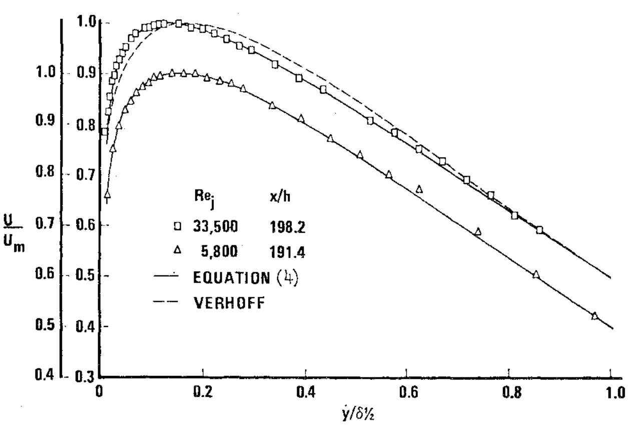


**Fig. 2 Velocity profile, $U_j/U_e = \infty$.**

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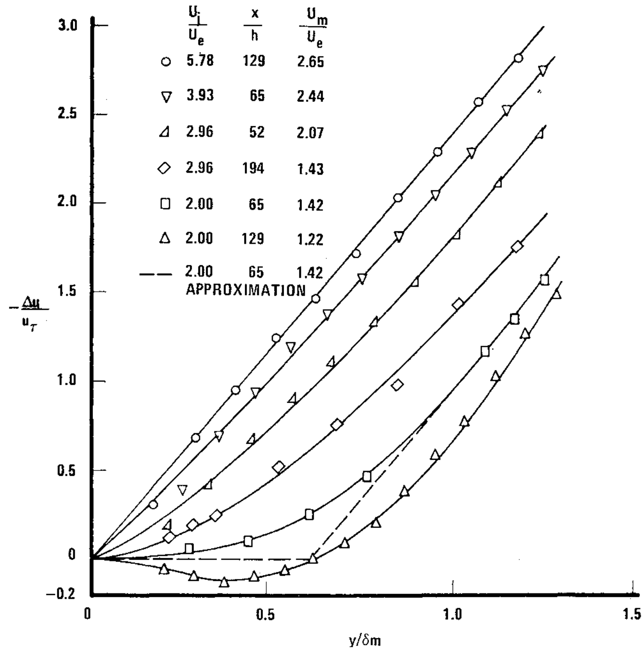


**Fig. 3 Velocity deviations from law of the wall.**

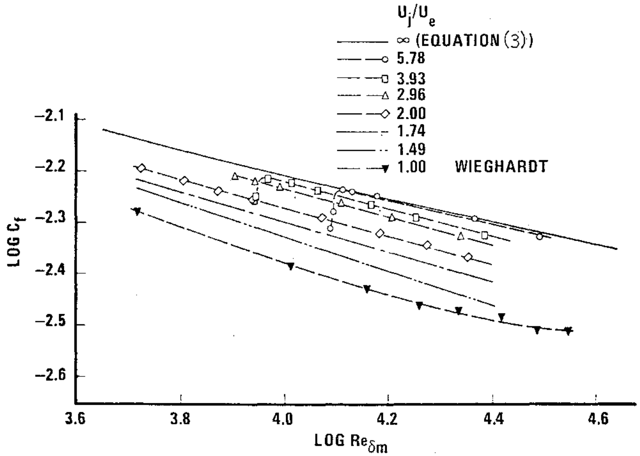


**Fig. 4 Friction coefficients.**

Figure 3 shows typical velocity deviations from Coles' law of the wall for finite $U_m/U_e$. For $U_m/U_e = 2.65$ the linear deviation in the wall layer is still evident and the preceding results for $U_m/U_e = \infty$ are applicable. At lower velocity ratios, the approach to the law of the wall is asymptotic.

$C_f$ curves determined from matching the velocity profiles to the law of the wall are presented in Fig. 4. The solid curve for $U_j/U_e = \infty$ is Eq. (3). The results for $U_j/U_e = 2.0$ have converged to those obtained from the Preston tube data. Consequently, the curves for the lower two velocity ratios are from the Preston tube data. The lower limit is established by Wieghardt's boundary-layer results. For $U_j/U_e = 3.93$ and 5.78, $C_f$ initially increases rapidly with $Re_{\delta_m}$. This is the region of transition from an initial turbulent boundary to a developed wall jet. This transition requires about 50 slot heights for all $U_j/U_e$.

The results of Fig. 4 can be represented in terms of local conditions by the empirical equation

$$\left(\frac{2}{C_f}\right)^{1/2} = 2.808 \log C_f + 5.616 \log Re_{\delta_m} + 6.244(\delta_m/\delta_{1/2}) + 0.838 \tag{5}$$

This equation characterizes the outer layer effect by the scale factor $\delta_m/\delta_{1/2}$ and is in close agreement with Eq. (3) for $U_j/U_e = \infty$. As $\delta_m/\delta_{1/2} \to 1$ this becomes the boundary-layer equation for Coles' $\Pi = 0.6$.

Wall layer shape parameters $\delta^*/\theta$ and $\theta/\delta_m$ ($\delta^*$ and $\theta$ are the displacement and momentum thicknesses) were also computed. The velocity profiles outside the sublayer were represented by Eq. (2) for $U_j/U_e = \infty$ and by linear approximations of the deviations from the law of the wall, as illustrated in Fig. 3, for finite $U_j/U_e$ (the approximations are expressed in terms of $\delta_{1/2}/\delta_m$). These results show that $\delta^*/\theta$ is essentially the same as that for a boundary layer with Coles' $\Pi = 0$ and considerably different from those for a power law profile with the fullness required to give the proper $\theta/\delta_m$. The wall layer shape parameters were also used to compute $C_f$ by the method of Thompson[2] for boundary layers. The computed values of $C_f$ differed by about 5% from the actual values at the low Reynolds numbers. The difference diminished with increasing Reynolds numbers.

## References


[1] Coles, D. E., and Hirst, E. A., "Computation of Turbulent Boundary Layers," *AFOSR—IFP—Stanford Conference*, Vol. 2. Stanford, University Press, Stanford, Calif., 1968.

[2] Head, M. R., and Patel, V. C., "Improved Entrainment Method for Calculating Turbulent Boundary Layer Development," R & M 3643, Aeronautical Research Council, London, England, 1969.